Synthesis of Configuration of Uniformly Radiating Longitudinal Slots in the Sections of Nonregular Rectangular Waveguides

Michael V. Davidovich and Valery P. Meschanov

***Abstract*—The integral-equation method has been applied along with perturbation theory to investigate the leaky-wave parameters of the fundamental mode in the regular waveguide with a narrow longitudinal slot on its broad wall, opened into shielded semispace. Then, the simple analytical formulas have been derived for the engineering synthesis of radiating elements (RE's), which are formed by a nonuniform waveguide section with a nonregular slot on its broad wall. The long slot was cut in the longitudinal direction along certain curved or several connected broken lines, forming small local angles with the waveguide axis. The synthesis of RE's by single-mode approximation has been carried out for the condition of good uniformity of radiating electrical-field distribution along the RE axis near the slot. Good agreement of theoretical and experimental results were obtained.**

I. Introduction

A considerable number of works [1]–[5], have been dedicated to the investigation of longitudinal slots in broad walls of rectangular waveguides. Most of these works discuss cases of single resonant slots parallel to the waveguide axis, compound resonant slots [4], or phased arrays of such slots.

The investigation of long (the order of a number of wavelengths and more) slots cut in longitudinal direction along certain curved or broken lines, forming small local angles with the waveguide axis, is of much interest. A slot cut along the straight line located at a small angle to the waveguide axis exhibits improved uniformity of the distribution of power radiated into the near area as compared to the similar slot parallel to the waveguide axis [5]. The uniform power distribution along the axis of the radiating element (RE) is an important factor in industrial basic microwave units for heating and processing of materials where a good heating uniformity (i.e., the electrical-field uniformity on large surfaces) is required. It is also essential for many other applications. The principle requirement for such an RE is the uniformity of the power radiated into the near area along the waveguide axis. More uniform radiation in the design [5] is achieved by compensation of power decrease along the guide (which is subjected to the exponential law in rectangular waveguide with longitudinal slot) via the slope of the slot toward its sidewall where the radiation is maximum. However, the slot cut along a straight line does not provide optimum characteristics for an RE. Such characteristics can be achieved by synthesis of the optimum slot configuration. Besides, an RE on a regular waveguide section does not exhibit high efficiency since a large part of the power goes to the guide load. In addition, such an RE has limited length determined by the point where the slot touches the narrow wall of the waveguide. That is why it is expedient to consider an RE having a longitudinal slot cut along a certain optimum curve in the broad wall of a rectangular waveguide section with the height decreasing smoothly along its length. The use of a number of such RE's located in one or two parallel planes can provide sufficient heating of a thin flat sample of a large surface area.

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Rigorous full-wave analysis of the RE under consideration is extremely complicated due to the open three-dimensional and noncoordinate nature of the problem. The effect of the sample and of the change of its dielectric properties in the process of heating should also be taken into account. Therefore, the derivation of the approximate relationships for the synthesis of the above RE is of practical interest.

The goal of this paper is to get the approximate relationships for the engineering synthesis of an RE in the form of a narrow long longitudinal slot in the broad wall of the rectangular waveguide. Such relationships have been obtained in this paper on the basis of single-mode approximation of the incident field. In order to obtain the characteristics of the slot radiation, we have considered a full-wave electrodynamic model for the infinite longitudinal slot in a regular rectangular waveguide with an infinitely thin metal screen at the broad wall. Such characteristics for a narrow slot have been obtained through use of the perturbation theory.

II. Problem Solution and Results

We will consider the slots cut in a broad wall of a rectangular waveguide so that the angle of their slope toward the waveguide axis may vary along the axis, yet remains small (of the order of several degrees). Due to this, the longitudinal electrical field at the slot is small as compared to the transverse one, and may be neglected. Therefore, we will take into consideration only the E_x-component of the electrical field at the slot, which is approximately equal to the transverse field induced by the surface currents at the broad waveguide wall, which are lateral to the slot and cross it. In order to improve RE matching and to increase its efficiency, we will discuss the possibility of smooth reduction of the guide height. The analysis will be carried out in the approximation of the incident field of the fundamental wave (without taking into account the diffraction effects) for a narrow slot when the ratio d/a of the cross dimension of the slot to the guide width is small.

A. Problem a

Let us consider a regular rectangular waveguide with the slot cut along its axis, the cross section of which is shown in Fig. 1. The slot radiates into the semispace limited by the screen in the form of a continuation of the broad waveguide wall. A sample in the form of an infinite plate made of dielectric material with losses may be located in the semispace. The model described above shows good correspondence to the real requirements to the microwave processing and heating. Let us find the attenuation constant α in such a waveguide when $d \ll a$. The dependence of the fields along the z-axis will be searched for in the form $e^{-j\cdot\gamma\cdot z}$, where

$$\begin{aligned} \gamma &= \gamma_{10} + \Delta\lambda \\ \gamma_{10} &= (k^2 - (\pi/a)^2)^{1/2} \end{aligned} \tag{1}$$

$k = \omega(\varepsilon_0 \cdot \mu_0)^{1/2}$ is the wavenumber of free space, and $|\Delta\gamma| \ll \gamma_{10}$. The complex propagation factor γ of the structure under consideration satisfies the admittance equation

$$\int_{x_0-d/2}^{x_0+d/2} \breve{K}(x, x') E_x(x')\, dx' = 0 \tag{2}$$

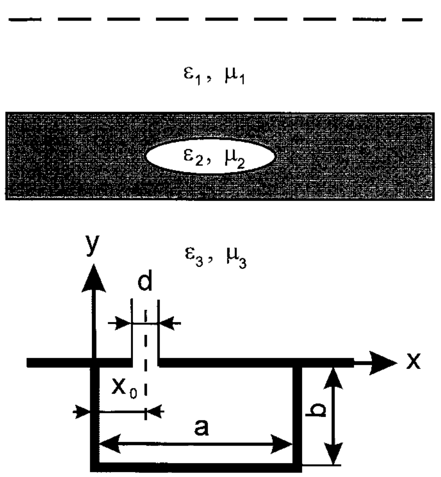


Fig. 1. A cross section of the infinite regular waveguide with a narrow longitudinal slot and three-layered medium with the sample located above the waveguide.

where the kernel has the form

$$\breve{K}(x,x') = \bar{K}(x,x') + \tilde{K}(x,x')$$
$$\bar{K}(x,x') = 2/a \sum_{n=0}^{\infty} \times \frac{\cos(n\pi x/a)\cdot\cos(n\pi x'/a)\cdot(k^2-\gamma^2)}{j\cdot\omega\cdot\mu_0\cdot(1+\delta_{n0})\cdot\chi_n\cdot tg(\chi_n b)},$$
$$\tilde{K}(x,x') = \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} d\kappa \frac{e^{-j\cdot\kappa(x-x')}}{\kappa^2+\gamma^2}\cdot\left(\frac{\kappa^2}{Z^e(\kappa)}+\frac{\gamma^2}{Z^h(\kappa)}\right). \quad (3)$$

Here, $\chi_n = [k^2-(n\pi/a)^2-\gamma^2]^{1/2}$, $Z^e(k)$, and $Z^h(\kappa)$ are E- and H-mode impedances of semispace for the wavenumber χ transformed to the slot plane [6]. In case the upper screen and dielectric samples are absent at the upper semispace, the impedances will have the form

$$Z^e(\kappa) = \frac{\sqrt{k^2-\kappa^2-\gamma^2}}{\omega\cdot\varepsilon_0}$$
$$Z^h(\kappa) = \frac{\omega\cdot\mu_0}{\sqrt{k^2-\kappa^2-\gamma^2}}. \quad (4)$$

The presence of such a screen leads to reactive impedances in (4). The presence of a multilayered medium is taken into account by recalculation of the impedances of the type (4) to the plane $y=0$ using the formula of their transformation sequentially for each layer. The admittance equation (2) may be obtained by conventional methods of field matching [6]–[9] through decomposition and representation of the fields inside the waveguide by Fourier series and outside the waveguide by Fourier integrals. If the fields in the upper semispace are not considered and the slot is simulated as an impedance surface with the impedance Z, then $\tilde{K}(x,x')$ in (2) should be replaced by $Z^{-1}\delta(x-x')$ [6], [9]. In this case, we will substitute the relationships (1) into (2) with the assumption in the slot E_x = constant, and expand in terms of the small parameter $\Delta\gamma/\gamma$. As a result, we get (5), shown at the bottom of the page, and

$$\Sigma = \left(\frac{\pi}{a}\right)^2 \sum_{n=2}^{\infty} \frac{\cos^2(n\pi x_0/a)\cdot S_n^2}{(n\pi/a)\cdot \mathrm{th}(n\pi b/a)}.$$

For the case of small d, these relationships can be easily transformed into

$$\Delta\gamma = \frac{j\cdot Z\cdot d\cdot(\pi/a)^2\cos^2(\pi x_0/a)}{k\cdot Z_0\cdot a\cdot b\cdot\gamma_{10}} \quad (6)$$

where $Z = (\mu_0/\varepsilon_0)^{1/2}$ is the free-space impedance. The use of the representation E_x = constant $\cdot\,(d^2-4(x-x_0)^2)$ satisfying the edge condition leads to the replacement of S_n by $S_n' = (\pi/2)J_0(n\pi d/(2a))$ with Bessel function J_0, and the relationship (6) being unchanged. Let us obtain now similar to (6) relationships taking into account the obvious form $\tilde{K}(x,x')$ for the impedances (4). Assuming the field at the slot constant and integrating the logarithmically singular function $\tilde{K}(x,x')$ under small ratio d/a, we find

$$\Delta\gamma = \frac{2j\cdot\cos^2(\pi x_0/a)}{a\cdot b\cdot\gamma_{10}\cdot\left(1+\frac{2j}{\pi}\cdot\ln\left(\frac{\bar{\gamma}_0\cdot\pi\cdot d}{2a}\right)\right)}. \quad (7)$$

Here, $\bar{\gamma}_0 = 1.781\,07\ldots$ is Euler's constant. The following expression for the attenuation constant α has been found:

$$\alpha = \pi^2\cdot\cos(\pi x_0/a)\cdot\left\{2a\cdot b\cdot\gamma_{10}\cdot\ln\left(\frac{\pi\cdot d\cdot\bar{\gamma}_0}{2a}\right)\right\}^{-1}. \quad (8)$$

B. Problem b

Let us assume further that x_0, b, and α are functions of the z-coordinate and find the optimum geometrical slot configuration on the condition that the amplitude of the transverse current induced by the fundamental wave is constant with respect to z. Such a condition has the form

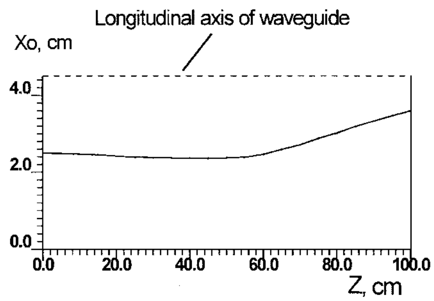
$$e^{-\alpha(z)z}\cdot\cos\left(\frac{\pi\cdot x_0(z)}{a}\right)\cdot(b_0/b(z))^{1/2} = \text{constant} = \cos\left(\frac{\pi\cdot x_1}{a}\right) \quad (9)$$

where $x_1 = x_0(0)$, L is the slot length, and $b_0 = b(0)$. The first multiplier corresponds to the reduction of the wave amplitude in the guide at the expense of the slot radiation, the second corresponds to the variation of the induced current due to the slot displacement with respect to the center of the broad wall, and the third to the increase of the wave amplitude caused by smooth reduction of the waveguide section height; i.e., to the increase of normalized wave conductivity of the fundamental wave in the nonuniform waveguide. Let us consider a number of individual cases. First, let the slot with constant dimension d be cut parallel to the waveguide axis. Then, from (9), it is easy to find the inverse dependence $z(b)$ having the form

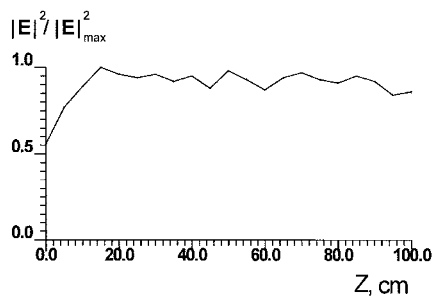
$$z(b) = \frac{1}{2\alpha}\cdot\ln(b_0/b). \quad (10)$$

More rapid decrease of height b with the growth of z as compared to the exponential law corresponds to the above dependence. In this case, the power leakage efficiency is of the order of 75%, and the height ratio is about $b_0/b(L) = e^2$ when the RE length $L = 1/\alpha$. This length, according to (8), increases inversely proportional to

$$\Delta\gamma = \frac{(\pi/a)^2\cos^2(\pi x_0/a)\cdot S_1^2}{b\gamma_{10}[2\cos(\pi x_0/a)\cdot S_1^2/b + (\pi/a)\cdot ctg(\pi b/a) + j\omega\mu_0 a/(dZ) - \Sigma]}$$
$$S_n = \frac{\sin\left(\frac{n\pi d}{2a}\right)}{\frac{n\pi d}{2a}} \quad (5)$$



(a)



(b)

Fig. 2. (a) Calculated slot profile configuration. (b) Measured electric-field squared (power) relative distribution along the axis for $L = 100$ cm, $h = 12$cm, and $d = 0.2$ cm.

$\sin^2(\pi\delta/a)$, where δ is the distance from the slot center to the central line of the broad wall. Now, let b be constant and x_0 vary. Then, z may be expressed via x_0 in the form

$$z(x_0) = \ln\left(\frac{\cos(\pi x_0/a)}{\cos(\pi x_1/a)}\right) \cdot (\alpha_m \cdot \cos^2(\pi x_0/a))^{-1}. \qquad (11)$$

Here, α_m is the maximum attenuation constant obtained from (8) for $x_0 = 0$. The maximum length L_m for the slot may be obtained from (11) for $x_0 = 0$. It is defined by the value x_1 and has the form $L_m = z(0) = -\ln(\cos(\pi x_1/a))/\alpha_m$. In order to provide the efficiency of approximately 50%, it is advantageous to choose this length from the condition $\alpha_m L_m > 2$. Let us now consider the case of an RE in the form of the tapered section of waveguide with length L and height dependence in the form $b(z) = b_0 + z(b_1 - b_0)/L$. Fig. 2 presents the numerical results of the synthesis of the slot profile (central line) according to the relationship (9) [Fig. 2(a)] and the measured results for the distribution of the electric field modulus along the axis at the distance $h = 12$ cm from the broad wall obtained with the use of the probe method [Fig. 2(b)]. The waveguide of dimensions 90 × 45 mm with $d = 2$ mm and $b_1/b_0 = 0.2$ has been used at the frequency of 2.45 GHz. The presence of the bend in the slot profile is explained by the fact that, at first, the inverse function derivative $b'(z)$ for the relationship (10) is smaller than the tangent of the slope angle $(b - b_0)/L$ of the linear function. To compensate for this, the slot slope with respect to the narrow wall is decreasing with the growth of z. These two values coincide at the middle range, and then $b'(z) > (b - b_0)/L$. The profile bend to the guide axis serves to compensate for this. The presence of a small dead zone at the beginning of the slot is attributed to the leaky character of the wave.

Smooth variation of the waveguide parameters along its length allows us to apply the theory of lines with continuously varying parameters [10], [11] in order to define the reflection and transfer coefficients, and on this basis to estimate the radiation efficiency. The complex propagation constant (1) leads to complex dominant mode impedance varying with z. Thus, for the reflection coefficient, we have

$$R = R_0 + \int_0^L \frac{\rho'(z)}{\rho(z)} \cdot e^{-2 \cdot j \cdot \gamma_{10} \cdot (L-z)} \cdot e^{\int_z^L \Delta\gamma(z')\,dz'} \cdot dz \qquad (12)$$

where $\rho(z) = b(z)/\{b_0(1 + \Delta\gamma(z)/\gamma_{10})\}$ is the normalized local waveguide impedance and $R_0 = -\Delta\gamma(0)/[2\gamma_{10} + \Delta\gamma(0)]$ is the reflection coefficient from a semiinfinite regular waveguide with a slot parallel to its axis and located at $x = x_1$. The radiating efficiency of an RE has been estimated by the application of this method. The correct consideration of the dielectric plate is also possible by the calculation of the residues and integrals through the cuts in the last mentioned relationship from (2). In order to divide the contribution of the surface and spatial waves into the real part of slot impedance or radiating capability obtained during this process, it is convenient to apply the procedure described in [12] dealing with the decomposition of the Green's function into longitudinal electric (LE)- and longitudinal magnetic (LM)- modes. It is more convenient to consider the upper region with side screens, which enables one to represent $\tilde{K}(x, x')$ in the form of a series. Both the theoretical evaluation and the experiment show strong dependence of the attenuation constants α from the parameters and configuration of the dielectric sample. However, there is no need to define the optimum slot configuration in each case. It is sufficient to do this for the most typical semispace medium configurations (e.g., when the dielectric filling is absent or for the standard sample). The variation of the sample parameters (e.g., change of e under drying) may be compensated by the change of the sample height above the slot or by introducing a displacement with respect to the height of the lossless dielectric insertion.

III. Conclusion

Simple analytical formulas for the engineering synthesis of the configuration of long longitudinal slot cut in a broad wall of the rectangular waveguide section and uniformly radiating into the near area have been obtained. Both regular and tapered forms of the waveguide section are presented. The formulas presented have been obtained in a single-mode approximation of the given incident field under the condition of a narrow slot, as compared to the dimension of the broad wall of the waveguide. The above formulas are based upon the properties of the regular uniform infinite waveguide with a longitudinal slot. The admittance integral equation has been used to define such parameters. The relationships obtained have been applied to the computation of the design of an RE and show good uniformity of power distribution in the near area, as confirmed by experiments.

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Mode-Matching Analysis of Circular-Ridged Waveguide Discontinuities

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***Abstract*— This paper describes a mode-matching algorithm for S-parameter computation of circular-ridged waveguide (CRW) discontinuities. The ridges are shaped like the cross section of a cone (pie-shaped) with a geometry that can be described in cylindrical coordinates. This idea avoids the use of a mixed-coordinate system in the analysis of the electromagnetic fields in the ridged sections, which can, therefore, be expressed in terms of modal functions. The resulting algorithm is fast and accurate and has been utilized to design and optimize a five-section double-ridge filter and a quadruple-ridge waveguide transformer. The measured response of the filter is in good agreement with the calculated data.**

***Index Terms*—Algorithm, circular waveguides, eigenvalue.**

I. Introduction

Circular-ridged waveguide (CRW) components like filters, polarizers, orthomode transducers, etc., are important elements in subsystems for satellite communications. Low-cost design, small size, and optimum performance of these components is essential to satisfy today's stringent payload requirements. In this context, ease of manufacturing and accurate computer-aided design of CRW components are equally important issues. This paper describes the design of CRW filters and transformers in which the ridge geometry has been modified from the usual rectangular shape to a pie shape (Fig. 1) for which the geometry can be described entirely in cylindrical coordinates. This approach avoids the use of a mixed-coordinate system in the field-theory analysis of CRW structures. Thus, simplifying the algorithm without complicating the fabrication of CRW components.

In [1] and [2], the authors have presented an eigenvalue analysis of pie-shaped ridge structures in circular waveguides by using the radial mode-matching method. In that work and due to the pie shaped ridges, the electromagnetic field in the subsections of the CRW were described by modal functions. On the basis of [1] and [2], a scattering parameter analysis of double-ridged circular waveguide discontinuities was presented in [3]. To verify the algorithm, we have designed, built, and tested a five resonator double-ridged filter and found very good agreement with theoretical data.

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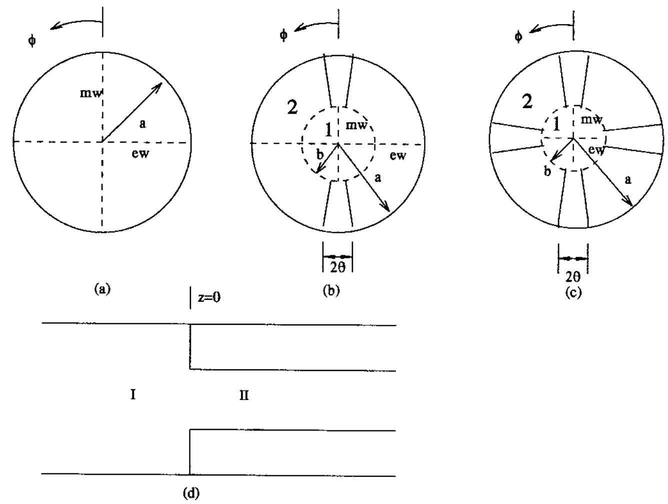

Fig. 1. Cross section of (a) circular waveguide (region I), (b) double-ridged circular waveguide (region II), (c) quadruple-ridged circular waveguide (region II), and (d) sideview of the discontinuity.

II. Theory

A mode-matching method is developed to calculate the generalized scattering matrix of a discontinuity between an empty circular waveguide and a double- or quadruple-ridged circular waveguide. The cross section of such a discontinuity is shown in Fig. 1. Region II in Fig. 1(d) can either be a double- or quadruple-ridged waveguide. Since the $\mathrm{TE}_{1,1}$ mode is the fundamental mode of propagation, a magnetic- and electric-wall symmetry can be used. The electric and magnetic potential functions in the empty circular waveguide for such a symmetry can be written as follows:

$$\psi^{(\mathrm{I}h)} = \sum_{m=1}^{M} \sum_{n=1,3}^{N} P_{n,m}^{\mathrm{I}h} J_n\left(k_{c_{n,m}}^{\mathrm{I}h}\rho\right) \sin n\phi \tag{1}$$

$$\psi^{(\mathrm{I}e)} = \sum_{m=1}^{M} \sum_{n=1,3}^{N} P_{n,m}^{\mathrm{I}e} J_n\left(k_{c_{n,m}}^{\mathrm{I}e}\rho\right) \cos n\phi. \tag{2}$$

The coefficients P represent the power normalization constants and are obtained by setting the magnitude of the power carried in each of the modes to unity. The eigenvalues of the circular waveguide for TE and TM modes can be determined from the zeros of the Bessel functions [4]. The values of M and N in (2) depend on the number of TE and TM modes used in the evaluation of the generalized scattering matrix. The electric and magnetic potential functions in the CRW (region II) can be written as a sum of those in subsections (1) and (2), shown in Fig. 1(b) or (c):

$$\psi^{(\mathrm{II}h)} = \sum_{r=1}^{R} \left(\psi^{(1h)} + \psi^{(2h)}\right) \tag{3}$$

$$\psi^{(\mathrm{II}e)} = \sum_{r=1}^{R} \left(\psi^{(1e)} + \psi^{(2e)}\right) \tag{4}$$